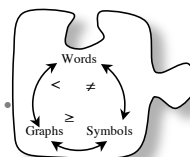


## 5.2.1 How do these compare?

### Comparing Expressions



You have been working with writing and simplifying expressions that represent the steps of a number trick. As you wrote these expressions you learned that it was helpful to simplify them by combining like terms and removing zeros. In this lesson you and your teammates will use a tool for comparing expressions to determine if one expression is greater than the other or if they are equal, that is, equivalent ways of writing the same thing.

Remember that to represent expressions with algebra tiles, you will need to be very careful about how positive and negative are distinguished. To help understand the diagrams in the text, the legend showing the shading for  $+1$  and  $-1$  at right will be placed on every page. This model also represents a zero pair.



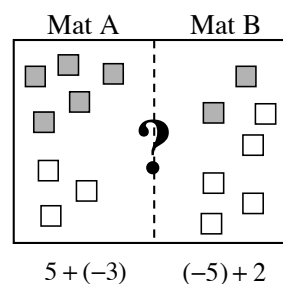
#### 5-40. COMPARING EXPRESSIONS

Ignacio and Olivia were playing a game. Each of them grabbed a handful of algebra tiles. They wanted to see whose expression had the greater value.

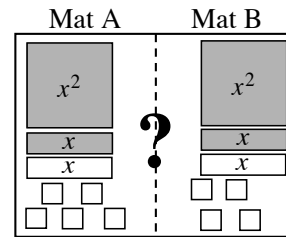
Two expressions can be compared by dividing the expression mat in half to change it into an **Expression Comparison Mat**. Then the two expressions can be built side by side and compared to see which one is greater.

- Olivia put her tiles on Mat A in the picture above and described it as  $5 + (-3)$ .
- Ignacio put his tiles on Mat B and said it was  $(-5) + 2$ .

With your team, find two different methods to simplify the two expressions in order to compare them. Which side of the mat is larger?



- 5-41. Using your Expression Comparison Mat, build the two expressions at right. Find a way to determine which side is greater, if possible. Show your work by sketching it on the Lesson 5.2.1B Resource Page. Be ready to share your conclusion and your **justification**.

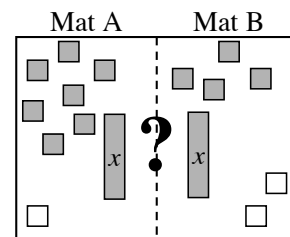


- 5-42. **MORE COMPARING EXPRESSIONS** – Is one expression greater?

Consider how you were able to compare the expressions in the previous problems. When is it possible to remove tiles to compare the expressions on the mats? In this problem you will work with your team to identify two different “legal moves” for simplifying expressions.



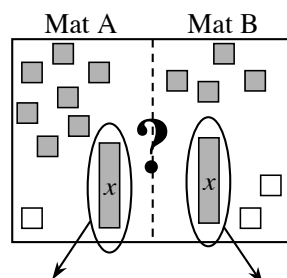
Build the mat at right using tiles and simplify the expressions. Record your work by drawing circles around the zeros or the balanced sets of tiles that you remove in each step on the Lesson 5.2.1B Resource Page. Which expression is greater?



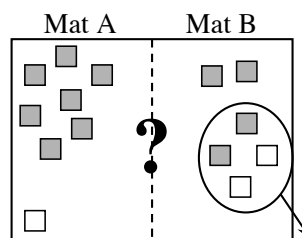
- 5-43. There are two kinds of moves you could use in problem 5-42 to simplify expressions with algebra tiles. They are removing zeros and removing matching (or balanced) sets of tiles from both sides of the mat as shown in the figures below. **Justify** why each of these moves can be used to simplify expressions.



**Removing Balanced Sets**



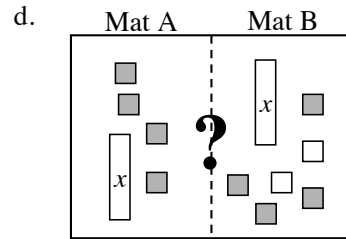
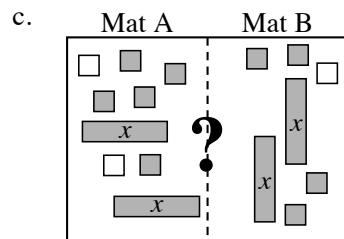
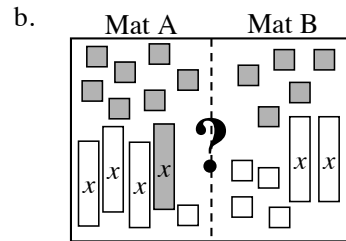
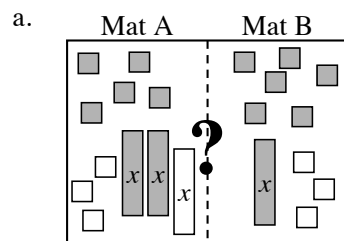
**Removing Zeros**



5-44. WHICH SIDE IS GREATER?

For each of the problems below, use the Lesson 5.2.1 Resource Page and:


- Build the two expressions on your mat.
- Write an expression for each side below the mats for parts (a) through (d).
- Draw the tiles in the space given on the resource page for parts (e) and (f).
- Use legal moves to determine which mat is greater, if possible. Record your work by drawing circles around the zeros or the balanced (matching) sets of tiles that you remove in each problem.



e. Mat A:  $3x - 4 - 2$   
Mat B:  $3(x - 1)$

f. Mat A:  $5 + (-3x) + 5x$   
Mat B:  $x^2 + 2x + 1 - x^2$

MATH NOTES



## METHODS AND MEANINGS

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### Inequality Symbols

Just as the symbol “=” is used to represent that two quantities are equal in mathematics, the **inequality symbols** at right are used when describing the relationships between quantities that are not necessarily equal. Examples:  $3 < 7$ ,  $14 \leq 14$ ,  $7 > 3$ ,  $19 \geq 14$ .

$<$  less than

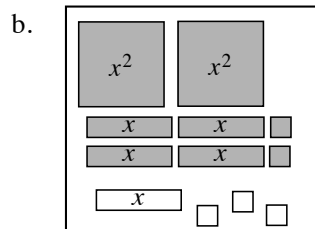
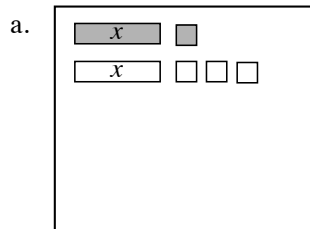
$\leq$  less than or equal to

$>$  greater than

$\geq$  greater than or equal to



- 5-45. Write the expression shown on each of the Expression Mats below, then simplify them by making zeros and combining like terms.



- 5-46. Copy and rewrite the following expressions by combining like terms, making zeros, and using the Distributive Property. **Visualizing** the expressions by using algebra tiles may be helpful.

a.  $(-3) + 4x + 2 + 2x + 2x$

b.  $-8x + 4 + (-3)$

c.  $7x^2 + 3x + 4 + 7x^2 + 3x + 4$

d.  $5(x - 4)$

5-47. Simplify the following expressions.

a.  $-\frac{3}{4} - \frac{2}{5}$

b.  $\frac{7}{8} - \frac{2}{3}$

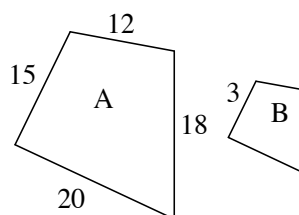
c.  $\frac{1}{3} - \frac{5}{6}$

d.  $1\frac{2}{3} + (-\frac{2}{5})$

e.  $\frac{4}{7} - (-\frac{3}{8})$

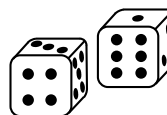
f.  $-4\frac{1}{2} + 3\frac{1}{9}$

5-48. Figures A and B at right are similar. Assuming that figure A is the original figure, find the scale factor and find the lengths of the missing sides of figure B.



5-49. Desmond is rolling a standard six-sided number cube. He plans to roll it 72 times.

a. About how many times would you expect Desmond to roll a 4? Why?



b. About how many times would you expect him to roll an even number? Why?

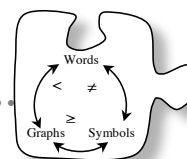
c. Desmond kept track of his results for all 72 rolls. The chart at right shows some of his results.

Based on his partial results, how many times did he roll a 5 or a 6?

Result	Number of outcomes
1	9
2	14
3	11
4	8

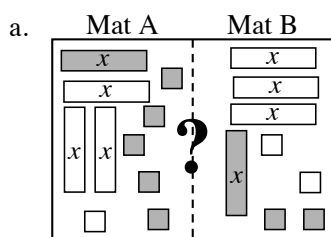
## 5.2.2 What if I can't tell?

### Comparing Quantities with Variables



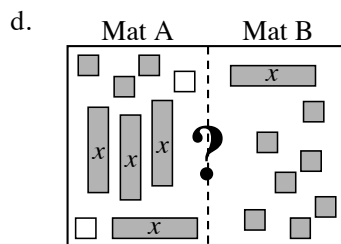
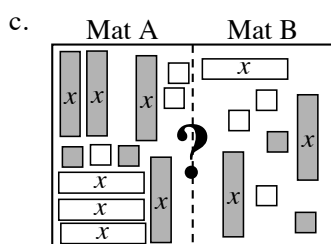
Have you ever tried to make a decision when the information you have is uncertain? Perhaps you have tried to make plans on a summer day only to learn that it *might* rain. In that case, your decision might have been based on the weather, such as, “I will go swimming if it doesn’t rain, or stay home and play video games if it does rain.” Sometimes in mathematics solutions might depend on something you do not know, like the value of the variable. Today you will study this kind of situation.

5-50. For each of the problems below, build the given expressions on your Expression Comparison Mat. Then use the simplification **strategies** of removing zeros and simplifying by removing matching pairs of tiles to determine which side is greater, if possible. Record your steps on the Lesson 5.2.2 Resource Page.



b.      Mat A:  $2(x + 3) - 4$

Mat B:  $3x + (-1) - x + 4$

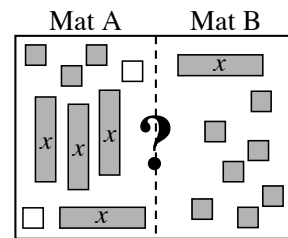


5-51. WHAT HAPPENED?

When Ignacio and Olivia compared the expressions in part (d) of problem 5-50, they could not figure out which side was greater.

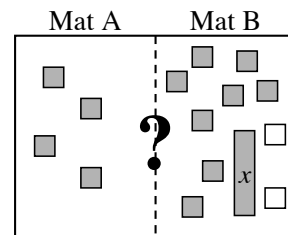
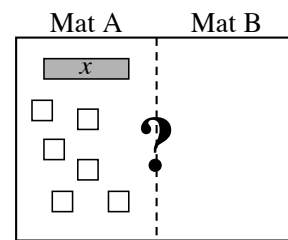


- Is it always possible to determine which side of the Expression Comparison Mat is greater (has the larger value)? Why or why not? Be prepared to share your **reasoning**.
- How is it possible for Mat A to have the greater value?
- How is it possible for Mat B to have the greater value?
- What other way can Mat A and B be related? Explain.



5-52. Ignacio and Olivia are playing another game with the algebra tiles. After they simplify two new expressions, they are left with the expressions on their mats shown at right. Can you tell by looking which part of the mat is greater?

- One way to compare is to separate the  $x$ -tiles and the unit tiles on different sides of the mat. Work with your team to find a way to have only  $x$ -tiles on Mat A. Make sure that you are able to **justify** that your moves are legal.
- Using the same **reasoning** from part (a), what would you do to have only variables on Mat B in the Expression Comparison Mat at right?
- Write a short note to Ignacio and Olivia explaining this new **strategy**. Feel free to give it a name so it is easier for them to remember.



- 5-53. Ignacio and Olivia are trying to decide if there are other ways to change expressions on the Expression Comparison Mat without affecting which side is greater. They have invented some new **strategies** and described them below.

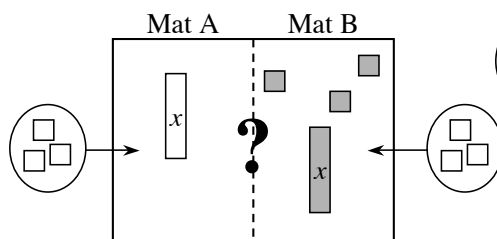


**Your task:** For each of the moves below:

- Build the Expression Comparison Mats on your paper.
- Follow each set of directions for the mat shown in each **strategy** below.
- Determine if the move in the **strategy** is valid for maintaining the relationship between the two expressions. Be prepared to **justify** your response.

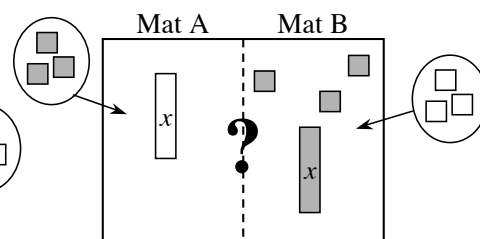
### Strategy #1

*“If you have a mat like the one drawn below, you can add the same number of tiles to both sides. In this case, I added 3 negative tiles to both sides.”*



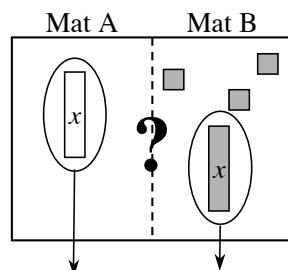
### Strategy #2

*“On a mat like the one below, I added +3 to Mat A and added -3 to Mat B.”*



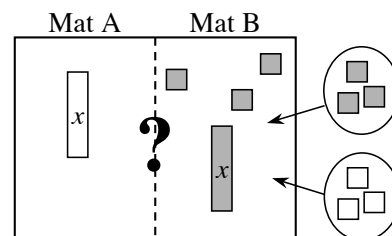
### Strategy #3

*“To simplify, I removed a positive x-tile from one side and a negative x-tile from the other side.”*



### Strategy #4

*“On a mat like the one below, I would add three zero pairs to Mat B.”*





5-54. LEARNING LOG

In your Learning Log, summarize the methods that your team and class developed to simplify expressions on the Expression Comparison Mat. Label your Learning Log entry “Simplifying Expressions (Legal Moves)” and include today’s date.



MATH NOTES

## METHODS AND MEANINGS

### Additive Identity and Additive Inverse

Formal properties that state some of the legal moves that you have been using are listed below.

**Additive Identity:** The number 0 is the additive identity because zero added to any number is equal to that number. The value is unchanged. For example,  $5.2 + 0 = 5.2$  and  $x + 0 = x$ .

**Additive Inverse:** The sum of a number and its opposite is equal to 0. That is, the additive inverse of a number is what needs to be added to the number to get a sum of 0.

Examples: For 2,  $-2$  is the additive inverse and  $2 + (-2) = 0$ .  
 For  $-5$ , 5 is the additive inverse and  $-5 + 5 = 0$ .  
 For  $x$ ,  $x + (-x) = 0$ .

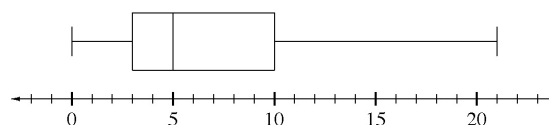


5-55. Simplify the expressions below.

- |                                     |                               |                           |
|-------------------------------------|-------------------------------|---------------------------|
| a. $5^2 \cdot (-3) - 4 \cdot 6 + 7$ | b. $-3 \cdot (6 + 4 \cdot 2)$ | c. $9 + 8 \div (-4) - 12$ |
| d. $2^3 - 3 \cdot 4 + 6(-1 + 2)$    | e. $4 + (3 + 4)^2$            | f. $\frac{8-13}{10}$      |

- 5-56. Write the following expressions in two ways, one with parentheses and one without. For example,  $4(x - 3)$  can be written  $4x - 12$ .
- A number reduced by 3, then multiplied by 2.
  - A number increased by 7, then multiplied by 5.
  - Ten times a number, then add twenty.
- 5-57. Graph these points on a coordinate grid:  $A(-2, 0)$ ,  $B(0, 4)$ ,  $C(4, 1)$ ,  $D(2, -3)$ . Connect the points in order, with point  $D$  connected to point  $A$ . What shape have you created?
- 5-58. Alan was paying a dinner check, but he was not sure how much he should tip for his bill of \$27.38. If a 15% tip is standard, about how much should Alan leave for the server?
- 5-59. Consider what you know about circles as you answer the questions below.
- What do you know about the lengths of the spokes of a bicycle wheel? Draw a picture of a bicycle wheel with spokes and explain what you mean.
  - As a bicycle travels on flat ground, how does the center of its wheels (the hub) move? Explain and draw a picture showing what you mean.
- 5-60. Elvin found the box-and-whisker plot below in the school newspaper.

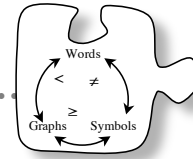
**Number of hours spent watching TV each week**



- Based on the plot, what percent of students watch more than 10 hours of television each week?
- Based on the plot, what percent of students watch less than 5 hours of television each week?
- Can Elvin use the box-and-whisker plot to find the mean (average) number of hours of television students watch each week? If so, what is it? Explain your **reasoning**.
- Can he use the plot to find the median? If so, what is it? Explain your **reasoning**.

## 5.2.3 Where do the solutions begin and end?

### One Variable Inequalities



You have used Expression Comparison Mats to compare two expressions and found that sometimes it is possible to determine which expression is greater. You will again compare expressions, this time finding the values for the variable that make one expression greater than the other.



- 5-61. Maria has been recording her work to see which side of an Expression Comparison Mat is greater, but she has been called away. Garth looked at her work, but he cannot figure out what Maria did to get from one step to another.

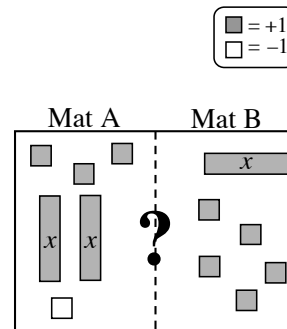
Mat A	Mat B
$5x + 2 + (-6)$	$2x + 2 + (-8)$
$5x + (-4)$	$2x + (-6)$
$3x + (-4)$	$-6$
$3x$	$-2$

Look at Maria's work above and help Garth by building the expressions on your mat and simplifying them. Write him a note explaining what Maria did to get from one step to another.

- 5-62. Use Maria's method of recording to show your steps while you compare the expressions  $2 + 2x + (-3)$  and  $2x + (-4) + 1$  using algebra tiles. Make sure you record each step so that your teacher can see what you did on your Expression Comparison Mat.
- Which mat is greater?
  - Use symbols (such as  $<$ ,  $=$ , or  $>$ ) to show the relationship of the final expressions on Mat A and Mat B.

- 5-63. Maria and Garth were playing a game with the algebra tiles. They each grabbed a handful of tiles and put them on the Expression Comparison Mat at right to see whose side had greater value.

Maria said, “*I have Mat A and my side has more value.*” Garth, who had Mat B, disagreed with her.



- Write expressions for Mat A and Mat B.
- Work with your team to simplify the expressions on the Expression Comparison Mat while carefully recording your work for each step on your paper. Can you tell whose side is greater? Why or why not?
- With your team, find at least four values for  $x$  that would make the expression on Maria’s side (Mat A) greater than the expression on Garth’s side (Mat B). Be prepared to share your values with the class.
- Any value for  $x$  that makes Mat A greater than Mat B is a solution to the inequality  $2x + 3 + (-1) > x + 5$ . This is read, “*Two  $x$  plus three plus negative one is greater than  $x$  plus five.*”

Share your solutions with another team and see if you have the same solutions that they do.

- 5-64. Karla had a hard time keeping track of all of the solutions to the inequality in problem 5-63 in her head. She decided to try to organize her answers. First she needed to know more about the problem.

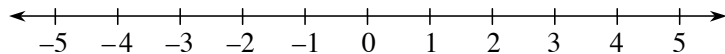
- Is there a greatest number that is a solution? Discuss this question with your team and be prepared to share your ideas with the class.
- Is there a smallest number that is a solution? Again, be prepared to share your team’s thinking with the class.



*Problem continues on next page. →*

5-64. *Problem continued from previous page.*

- c. What is special about the point where the solutions end? (This number is called the **boundary point**.) In other words, what relationship does this number have to the two expressions being compared?
- d. Karla was tired of listing so many solutions and wanted a way to show all of the solutions to this inequality quickly. She decided to draw a number line like the one below.



On your own paper, draw a number line such as the one above then follow your teacher's directions to represent the answer to this question on your number line.

5-65. Now consider the inequality  $2x + 5 < 3$ , which can be read “Two  $x$  plus five is less than 3.”



Build the inequality on your Expression Comparison Mat and record each step using variables on your paper. Work with your team to describe the least and greatest solutions to the inequality and draw your solution on a number line. Be prepared to **justify** your ideas.

5-66. Jerry and Ken were solving the inequality  $6 > 2x + 2$ . They set up the inequality on their Expression Comparison Mat and simplified it.

- a. Write a sentence in words to represent the original inequality.
- b. What did they get on each side of the mat when they simplified? Record your work on your paper.
- c. Graph all the solutions to this inequality on a number line.



- 5-67. Write an algebraic expression for each mat below. Then use the legal moves that you have developed to simplify each mat and, if possible, decide which expression is greater.

a.

b.

- 5-68. Graph these inequalities on a number line.

a.  $x > 3$                       b.  $x \leq 5$                       c.  $x \geq -4$

- 5-69. Lynn was shopping and found a purse that was marked with a discount of " $\frac{1}{3}$  off." If the original cost of the purse was \$80, how much will Lynn pay?

- 5-70. The girls' basketball team weighed each player. Their weights in pounds were 120, 122, 126, 130, 133, 147, 115, 106, 120, 112, and 142.

- Make a stem-and-leaf plot of the teams' weights.
- What is the median weight of girls on the team?
- What is the range of the data?
- What are the mean and mode?

- 5-71. Simplify each expression.

a.  $\frac{2}{3}(0.8)$                       b.  $\frac{4}{3} \cdot \frac{3}{7}$                       c.  $-\frac{5}{6} \cdot \frac{4}{7}$                       d.  $-\frac{4}{5} \cdot (-1\frac{1}{3})$